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14.18

$$f(x)=\begin{cases} e^x-e, & x<1 \\ \dfrac{\sqrt{\ln x}}{x}, & x\geq 1 \end{cases}$$

α) $\left.\begin{array}{l}\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-}(e^x-e) = e-e=0 \\ \lim\limits_{x\to 1^+} f(x) = f(1) = \dfrac{\sqrt{\ln 1}}{1} = 0\end{array}\right\} \Rightarrow \ldots$

β) C_f, $x'x$, $x=0$, $x=e$

$x<1$: $f(x)=e^x-e<0$ διότι $e^x : \uparrow \mathbb{R}$

$x>1$: $f(x)=\dfrac{\sqrt{\ln x}}{x}>0$

x	$-\infty$	0	1	e	$+\infty$
$f(x)$	$-$	$-$	0	$+$	$+$

$$E(\Omega)=\int_0^e |f(x)|\,dx = -\int_0^1 f(x)\,dx+\int_1^e f(x)\,dx =$$

$$= -\int_0^1 (e^x-e)\,dx + \int_1^e \frac{\sqrt{\ln x}}{x}\,dx =$$

$u=\ln x$

$du=\frac{1}{x}dx$

$u_1=\ln 1=0$

$u_2=\ln e=1$

$$= -\left[e^x-ex\right]_0^1 + \int_0^1 \sqrt{u}\,du =$$

$$= -\cancel{e} + \cancel{e} + 1 + \left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 = 1 + \left[\frac{2u\sqrt{u}}{3}\right]_0^1 =$$

$$= 1 + \frac{2}{3} = \frac{5}{3} \quad \text{τ.μ.}$$