Ασκήσεις : 13.31, 13.32, 13.35, 13.36, 13.39 (δ)

12/01/2026

13.27

$x_1, x_2 : \alpha x^2 + \beta x + \gamma = 0$, $\alpha, \beta, \gamma \neq 0$

$\beta + \gamma = 0$, $\frac{1}{x_1} + \frac{1}{x_2} = ;$

$$S = x_1 + x_2 = -\frac{\beta}{\alpha}, \quad P = x_1 \cdot x_2 = \frac{\gamma}{\alpha}$$

$$\frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1 + x_2}{x_1 \cdot x_2} = \frac{-\frac{\beta}{\alpha}}{\frac{\gamma}{\alpha}} = -\frac{\beta}{\gamma} = -(-1) = 1$$

13.30

α) -3 και 5

$S = -3 + 5 = 2$

$P = -3 \cdot 5 = -15$

άρα $x^2 - Sx + P = 0 \Longrightarrow x^2 - 2x - 15 = 0.$

β) $\frac{1}{2}$ και -2

$S = \frac{1}{2} - 2 = \frac{1}{2} - \frac{4}{2} = -\frac{3}{2}$

$P = \frac{1}{2} \cdot (-2) = -\frac{2}{2} = -1$

άρα $x^2 - Sx + P = 0 \Leftrightarrow x^2 + \frac{3}{2}x - 1 = 0 \longrightarrow 2x^2 + 3x - 2 = 0.$

13.33

α) $|x-2| = \sqrt{3} \Rightarrow$

$x - 2 = \sqrt{3}$ ή $x - 2 = -\sqrt{3} \Rightarrow$

$\boxed{x = 2 + \sqrt{3}}$ ή $\boxed{x = 2 - \sqrt{3}}$

$|x| = \vartheta \Leftrightarrow x = \vartheta$ ή $x = -\vartheta$

β) $2-\sqrt{3}$, $2+\sqrt{3}$

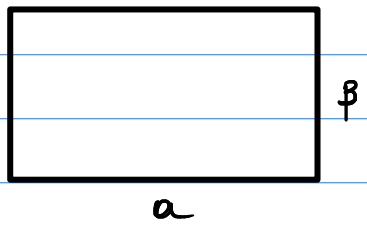
$S = 2-\sqrt{3}+2+\sqrt{3} = 4$

$P = (2-\sqrt{3})(2+\sqrt{3}) = 2^2-(\sqrt{3})^2 = 4-3 = 1$

Άρα $x^2 - Sx + P = 0 \Longrightarrow x^2 - 4x + 1 = 0$.

13.34 |

Ορθογώνιο με $\Pi = 20$ cm και $E = 24$ cm^2



α) $\Pi = 20 \Longrightarrow$ $E = 24 \Longrightarrow$

$2\alpha + 2\beta = 20 \Longrightarrow$ $\alpha \cdot \beta = 24 \Longrightarrow$

$\alpha + \beta = 10 \Longrightarrow$ $P = 24$

$S = 10$

Άρα $x^2 - Sx + P = 0 \Longrightarrow x^2 - 10x + 24 = 0$.

β) • $x^2 - 10x + 24 = 0 \Longrightarrow$

$(x-6)(x-4) = 0 \Longrightarrow$

$x = 6$ ή $x = 4$

α β

• $x^2 - 10x + 24 = 0$

$\Delta = (-10)^2 - 4 \cdot 24 = 100 - 96 = 4$

$x_{1,2} = \frac{10 \pm 2}{2} = \begin{cases} 6 = \alpha \\ 4 = \beta \end{cases}$

13.37 |

$x^2 - kx - 2$, $k \in \mathbb{R}$

α) $\Delta = (-k)^2 - 4 \cdot (-2) = k^2 + 8$

$\forall\, k \in \mathbb{R}: k^2 \geq 0 \Longrightarrow k^2 + 8 \geq 8$, άρα $\Delta > 0$

β) $x_1, x_2 : x^2 - 3x - 2 = 0$

i) $S = x_1 + x_2 = -\frac{-3}{1} = 3$ και $P = x_1 \cdot x_2 = \frac{c}{a} = \frac{-2}{1} = -2$

ii) $\rho_1 = 2x_1$, $\rho_2 = 2x_2$

$S' = \rho_1 + \rho_2 = 2x_1 + 2x_2 = 2(x_1 + x_2) = 2 \cdot 3 = 6$
$P' = \rho_1 \cdot \rho_2 = 2x_1 \cdot 2x_2 = 4x_1x_2 = 4 \cdot (-2) = -8$
Άρα $x^2 - S'x + P' = 0 \Longrightarrow x^2 - 6x - 8 = 0.$

13.39

x_1, x_2 : ρίζες της $-x^2 + x + 3 = 0 \Longrightarrow x^2 - x - 3 = 0$, άρα $\begin{cases} x_1 + x_2 = 1 \\ x_1x_2 = -3 \end{cases}$

α) $x_1^2x_2$ και $x_1x_2^2$

$S = x_1^2x_2 + x_1x_2^2 = x_1x_2(x_1 + x_2) = -3 \cdot 1 = -3$
$P = x_1^2x_2 \cdot x_1x_2^2 = x_1^3 \cdot x_2^3 = (x_1 \cdot x_2)^3 = (-3)^3 = -27$
Άρα $x^2 - Sx + P = 0 \Longrightarrow x^2 + 3x - 27 = 0.$

β) $\frac{x_1}{x_2}$ και $\frac{x_2}{x_1}$

$S = \frac{x_1}{x_2} + \frac{x_2}{x_1} = \frac{x_1^2 + x_2^2}{x_1x_2} = \frac{(x_1 + x_2)^2 - 2x_1x_2}{x_1x_2} = \frac{1^2 - 2 \cdot (-3)}{-3} = -\frac{7}{3}$

$P = \frac{x_1}{x_2} \cdot \frac{x_2}{x_1} = 1$
Άρα $x^2 - Sx + P = 0 \Longrightarrow x^2 + \frac{7}{3} \cdot x + 1 = 0 \Longrightarrow 3x^2 + 7x + 3 = 0.$

γ) $\frac{1}{x_1 - 1}$ και $\frac{1}{x_2 - 1}$

$S = \frac{1}{x_1 - 1} + \frac{1}{x_2 - 1} = \frac{x_2 - 1 + x_1 - 1}{(x_1 - 1)(x_2 - 1)} = \frac{x_1 + x_2 - 2}{x_1x_2 - (x_1 + x_2) + 1} =$

$= \frac{1 - 2}{-3 - 1 + 1} = \frac{-1}{-3} = \frac{1}{3}$
$P = \frac{1}{x_1 - 1} \cdot \frac{1}{x_2 - 1} = \frac{1}{-3} = -\frac{1}{3}$
Άρα $x^2 - Sx + P = 0 \Longrightarrow x^2 - \frac{1}{3}x - \frac{1}{3} = 0 \Longrightarrow 3x^2 - x - 1 = 0$