20/01/2026

2.38|

$f: [-\pi, 2\pi] \to \mathbb{R}$

$f(x) = 12(2x-1)\eta\mu x - 12(x^2 - x - 2)\sigma\upsilon\nu x - 2x^3 + 3x^2$

$$\ldots\ f'(x) = 24\eta\mu x + \cancel{12(2x-1)\sigma\upsilon\nu x} - \cancel{12(2x-1)\sigma\upsilon\nu x} + 12(x^2-x-2)\eta\mu x - 6x^2 + 6x =$$

$$= 12(x^2-x)\eta\mu x - 6(x^2-x) = 6(x^2-x)(2\eta\mu x - 1)$$

- $x^2 - x = 0 \Rightarrow x = 0$ ή $x = 1$
- $2\eta\mu x - 1 = 0 \Rightarrow \eta\mu x = \frac{1}{2} \xrightarrow{x \in [-\pi, 2\pi]} x = \frac{\pi}{6}$ ή $x = \frac{5\pi}{6}$

x	$-\pi$		0		$\frac{\pi}{6}$		1		$\frac{5\pi}{6}$		2π
x^2-x		$+$	0	$-$		$-$	0	$+$		$+$	
$2\eta\mu x-1$		$-$		$-$	0	$+$		$+$	0	$-$	
f'		$-$	0	$+$	0	$-$	0	$+$	0	$-$	
f		↘		↗		↘		↗		↘	

- $2\eta\mu 0 - 1 = -1 < 0$
- $2\eta\mu \frac{\pi}{2} - 1 = 1 > 0$
- $2\eta\mu \pi - 1 = -1 < 0$

2.42|

$$f(x) = \begin{cases} x^2 - 4x, & x < 3 \\ -x^2 + 10x - 24, & x \geq 3 \end{cases}$$

α) $$\left.\begin{aligned} &\lim_{x\to3^-} f(x) = \lim_{x\to3^-}(x^2-4x) = 9 - 12 = -3 \\ &\lim_{x\to3^+} f(x) = f(3) = -9 + 30 - 24 = -9 + 6 = -3 \end{aligned}\right\} \Longrightarrow \ldots\ f: \text{συνεχής στο } 3$$

β) $$\left.\begin{aligned} &\lim_{x\to3^-} \frac{f(x)-f(3)}{x-3} = \lim_{x\to3^-} \frac{x^2-4x+3}{x-3} = \lim_{x\to3^-} \frac{\cancel{(x-3)}(x-1)}{\cancel{x-3}} = 3 - 1 = 2 \\ &\lim_{x\to3^+} \frac{f(x)-f(3)}{x-3} = \lim_{x\to3^+} \frac{-x^2+10x-21}{x-3} = \lim_{x\to3^+} \frac{\cancel{(x-3)}(-x+7)}{\cancel{x-3}} = -3 + 7 = 4 \end{aligned}\right\} \Longrightarrow$$

$\Longrightarrow \ldots\ f$ δεν είναι παραγωγίσιμη στο 3.

γ) • $\underline{x<3}$

$\ldots\ f'(x) = (x^2-4x)' = 2x-4$

$f'(x)=0 \Rightarrow 2x-4=0 \Rightarrow x=2$

x	$-\infty$		2		3		$+\infty$
$2x-4$		$-$	0	$+$		$+$	

• $\underline{x>3}$

$\ldots\ f'(x) = (-x^2+10x-24)' = -2x+10$

$f'(x)=0 \Rightarrow -2x+10=0 \Rightarrow x=5$

x	$-\infty$		3		5		$+\infty$
$-2x+10$		$+$		$+$	0	$-$	

Άρα $f'(x) = \begin{cases} 2x-4, & \text{αν } x<3 \\ -2x+10, & \text{αν } x>3 \end{cases}$

x	$-\infty$		2		3		5		$+\infty$
f'		$-$	0	$+$		$+$	0	$-$	
f		$\searrow$		$\nearrow$		$\nearrow$		$\searrow$	

2.45

$$f(x) = \begin{cases} x^3-12x+5, & \text{αν } x\leq 1 \\ 2x\ln x-6x, & \text{αν } x>1 \end{cases}$$

$$\left.\begin{array}{l} \lim\limits_{x\to 1^-} f(x) = f(1) = 1-12+5 = -6 \\ \lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+}(2x\ln x-6x) = 2\ln 1-6 = -6 \end{array}\right\} \longrightarrow \ldots\ f \text{ συνεχής στο } 1$$

$$\lim_{x\to 1^-}\frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^-}\frac{x^3-12x+11}{x-1} = \lim_{x\to 1^-}\frac{\cancel{(x-1)}(x^2+x-11)}{\cancel{x-1}} = -9$$

1	0	-12	11	1
$\downarrow$	1	1	-11	
1	1	-11	$\boxed{0}$	

$$\lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^+}\frac{2x\ln x-6x+6}{x-1} = \lim_{x\to 1^+}\left(\frac{2x\ln x}{x-1}-\frac{6\cancel{(x-1)}}{\cancel{x-1}}\right) = 2-6=-4, \text{ διότι:}$$

• $\lim\limits_{x\to 1^+}\dfrac{2x\ln x}{x-1} = \lim\limits_{x\to 1^+}\left(2x\cdot\dfrac{\ln x}{x-1}\right) = 2\cdot 1 = 2$, διότι:

• $\lim\limits_{x\to 1^+}\dfrac{\ln x}{x-1} \underset{DLH}{\overset{\frac{0}{0}}{=}} \lim\limits_{x\to 1^+}\dfrac{1}{x} = 1$

De L' Hospital (DLH)

$\frac{0}{0}$, $\frac{\pm\infty}{\pm\infty}$

$$\lim_{x\to x_0}\frac{f(x)}{g(x)} = \lim_{x\to x_0}\frac{f'(x)}{g'(x)}$$

$x_0^+, x_0^-, +\infty, -\infty$

Άρα f δεν είναι παραγωγίσιμη στο 1

- $x<1$

... $f'(x)=(x^3-12x+5)'=3x^2-12$

$f'(x)=0 \Rightarrow x^2=4 \Rightarrow x=2$ ή $x=-2$
(απορ.)

x	$-\infty$		-2		1		2		$+\infty$
$3x^2-12$		$+$	0	$-$		$-$	0	$+$	

- $x>1$

... $f'(x)=(2x\ln x-6x)'=2\ln x+2-6=2\ln x-4$

$f'(x)=0 \Rightarrow \ln x=2 \Longrightarrow x=e^2$

x	0		1		e^2		$+\infty$
$2\ln x-4$		$-$		$-$	0	$+$	

- $2\ln 1-4=-4<0$
- $2\ln e^3-4=6-4=2>0$

$$f'(x)=\begin{cases}3x^2-12, & \alpha\nu\ x<1\\ 2\ln x-4, & \alpha\nu\ x>1\end{cases}$$

x	$-\infty$		-2		1		e^2		$+\infty$
f'		$+$	0	$-$		$-$	0	$+$	
f		↗		↘		↘		↗	

Άρα $f: \downarrow [-2, e^2]$.

2.54

$f(x) = 2e^{x-1} - x^2 + 3 \ , \ x \in \mathbb{R}$

$\ldots \quad f'(x) = 2e^{x-1} - 2x \ , \ x \in \mathbb{R}$

$\ldots \quad f''(x) = 2e^{x-1} - 2 \ , \ x \in \mathbb{R}$

$f''(x) = 0 \implies e^{x-1} = 1 \implies e^{x-1} = e^0 \xrightarrow{e^x : 1-1} x - 1 = 0 \implies x = 1$

x	$-\infty$		1		$+\infty$
f''		$-$	0	$+$	
f'		$\searrow$	O.E.	$\nearrow$	

• $f''(0) = \frac{2}{e} - 2 = \frac{2-2e}{e} < 0$

• $f''(2) = 2e - 2 = 2(e-1) > 0$

$$f'(1) = 2e^0 - 2 = 2 - 2 = 0$$

$\forall \ x \in \mathbb{R} : f'(x) \geq f'(1) \implies f'(x) \geq 0$, άρα $f : \uparrow \mathbb{R}$

2.53

$$f(x) = -\frac{2 + x^2 e^x + 2xe^x}{2e^x} \ , \ x \in \mathbb{R}$$

$\forall \ x \in \mathbb{R} : \ f(x) = -\frac{\cancel{2}}{\cancel{2}e^x} - \frac{x^2 \cancel{e^x}}{2\cancel{e^x}} - \frac{\cancel{2}x\cancel{e^x}}{\cancel{2}\cancel{e^x}} = -e^{-x} - \frac{x^2}{2} - x .$

$\ldots \quad f'(x) = \left(-e^{-x} - \frac{x^2}{2} - x\right)' = e^{-x} - x - 1 \ , \ x \in \mathbb{R}$

$\ldots \quad f''(x) = -e^{-x} - 1 < 0 \ \forall \ x \in \mathbb{R} \implies f' : \downarrow \mathbb{R}$

με $f'(0) = e^0 - 1 = 0$

• $x < 0 \xRightarrow{f' : \downarrow} f'(x) > f'(0) \implies f'(x) > 0$ άρα $f : \uparrow (-\infty, 0]$

• $x > 0 \xRightarrow{f' : \downarrow} f'(x) < f'(0) \implies f'(x) < 0$ άρα $f : \downarrow [0, +\infty)$

2.62

$f(x) = x^2 + 2e^{x+1}, \quad x \in \mathbb{R}$

$\ldots \quad f'(x) = 2x + 2e^{x+1}, \quad x \in \mathbb{R}$

$\ldots \quad f''(x) = 2 + 2e^{x+1} > 0 \quad \forall x \in \mathbb{R} \Longrightarrow f' : \uparrow \mathbb{R}$

με $f'(-1) = -2 + 2e^0 = -2 + 2 = 0.$

- $x < -1 \xRightarrow{f' : \uparrow} f'(x) < f'(-1) \Longrightarrow f'(x) < 0 \Longrightarrow f : \downarrow (-\infty, -1]$
- $x > -1 \xRightarrow{f' : \uparrow} f'(x) > f'(-1) \Longrightarrow f'(x) > 0 \Longrightarrow f : \uparrow [-1, +\infty)$

x	$-\infty$		-1		$+\infty$
f'		$-$	0	$+$	
f	$+\infty$	$\searrow$	3	$\nearrow$	$+\infty$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} (x^2 + 2e^{x+1}) \xlongequal{(+\infty)+0} +\infty$$

$$f(-1) = (-1)^2 + 2e^0 = 1 + 2 = 3$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} (x^2 + 2e^{x+1}) \xlongequal{(+\infty)+(+\infty)} +\infty$$

Άρα $f(\mathbb{R}) = [3, +\infty)$.

2.41, 2.44, 2.49, 2.52, 2.55, 2.56, 2.57, 2.60, 2.61