

19/01/2026

13.77

β) $S = \frac{\lambda^2 + 1}{\lambda}$, $p = 1$, $\lambda \neq 0$

γ) i) $\lambda < 0$ άρα $S = \frac{\lambda^2 + 1}{\lambda} \xrightarrow[\lambda']{\frac{+}{-}} S < 0$
 $\left. \begin{matrix} S < 0 \\ p > 0 \end{matrix} \right\} \Rightarrow x_1, x_2 < 0$

ii) $|x_1 + x_2| \geq 2x_1x_2 \Rightarrow$

$|S| \geq 2p \xrightarrow{S < 0}$

$-S \geq 2p \Rightarrow$

$-\frac{\lambda^2 + 1}{\lambda} \geq 2 \xrightarrow{\lambda < 0}$

$-(\lambda^2 + 1) \leq 2\lambda \Rightarrow$

$-\lambda^2 - 1 \leq 2\lambda \Rightarrow$

$\lambda^2 + 2\lambda + 1 \geq 0 \Rightarrow$

$(\lambda + 1)^2 \geq 0$ που ισχύει $\forall \lambda < 0$.

14.9

α) $2x^2 - |x| - 6 = 0 \Rightarrow$

$2|x|^2 - |x| - 6 = 0 \xrightarrow{y = |x|}$

$2y^2 - y - 6 = 0.$

$\Delta = (-1)^2 - 4 \cdot 2 \cdot (-6) = 1 + 48 = 49 > 0, \dots$

$y_{1,2} = \frac{1 \pm 7}{4} = \begin{cases} \frac{8}{4} = 2 \\ \frac{-6}{4} = -\frac{3}{2} \end{cases}$

Άρα $y = 2$ ή $y = -\frac{3}{2} \Rightarrow$

$|x| = 2$ ή $|x| = -\frac{3}{2} \Rightarrow$

$x = 2$ ή $x = -2$ (δυνατά).

$x^2 = |x|^2$

$$\begin{aligned}
 \text{g)} \quad & (x-2)^2 - 4\sqrt{x^2-4x+4} + 3 = 0 \Rightarrow \\
 & (x-2)^2 - 4\sqrt{(x-2)^2} + 3 = 0 \Rightarrow \\
 & |x-2|^2 - 4|x-2| + 3 = 0 \xrightarrow{w=|x-2|} \\
 & w^2 - 4w + 3 = 0 \xrightarrow[S=4]{P=3} \\
 & w=1 \text{ \textit{η}} w=3 \Rightarrow \\
 & |x-2|=1 \text{ \textit{η}} |x-2|=3 \Rightarrow \\
 & x-2=1 \text{ \textit{η}} x-2=-1 \text{ \textit{η}} x-2=3 \text{ \textit{η}} x-2=-3 \Rightarrow \\
 & x=3 \text{ \textit{η}} x=1 \text{ \textit{η}} x=5 \text{ \textit{η}} x=-1.
 \end{aligned}$$

$$\sqrt{a^2} = |a|$$

ϕ/βελ. 232

$$\begin{aligned}
 \text{a)} \quad & x^4 - 2x^2 - 8 = 0 \xrightarrow{y=x^2} \\
 & y^2 - 2y - 8 = 0 \xrightarrow[S=2]{P=-8} \\
 & y=4 \text{ \textit{η}} y=-2 \Rightarrow \\
 & x^2=4 \text{ \textit{η}} x^2=-2 \Rightarrow \\
 & \quad \quad \quad (\alpha\delta\upsilon\sigma\mu) \\
 & |x|=2 \Rightarrow \\
 & x=2 \text{ \textit{η}} x=-2.
 \end{aligned}$$

$$\begin{aligned}
 \text{β)} \quad & x^6 + 7x^3 - 8 = 0 \xrightarrow{w=x^3} \\
 & w^2 + 7w - 8 = 0 \xrightarrow[S=-7]{P=-8} \\
 & w=1 \text{ \textit{η}} w=-8 \Rightarrow \\
 & x^3=1 \text{ \textit{η}} x^3=-8 \Rightarrow \\
 & x=1 \text{ \textit{η}} x=-2.
 \end{aligned}$$

14.11

$$\begin{aligned}
 \alpha) \quad & x - 6\sqrt{x} + 8 = 0 \xrightarrow{x \geq 0} \\
 & (\sqrt{x})^2 - 6\sqrt{x} + 8 = 0 \xrightarrow{y=\sqrt{x}} \\
 & y^2 - 6y + 8 = 0 \xrightarrow[S=6]{P=8} \\
 & y=2 \text{ \textit{η}} y=4 \Rightarrow \\
 & \sqrt{x}=2 \text{ \textit{η}} \sqrt{x}=4 \Rightarrow \\
 & x=4 \text{ \textit{η}} x=16
 \end{aligned}$$

$$\begin{aligned}
 \text{β)} \quad & x - 3\sqrt{x} - 10 = 0 \xrightarrow{x \geq 0} \\
 & (\sqrt{x})^2 - 3\sqrt{x} - 10 = 0 \xrightarrow{w=\sqrt{x}} \\
 & w^2 - 3w - 10 = 0 \xrightarrow[S=3]{P=-10} \\
 & w=5 \text{ \textit{η}} w=-2 \Rightarrow \\
 & \sqrt{x}=5 \text{ \textit{η}} \sqrt{x}=-2 \Rightarrow \\
 & \quad \quad \quad (\alpha\delta\upsilon\sigma\mu) \\
 & x=25
 \end{aligned}$$

$$8) \quad x + 5\sqrt{x} + 4 = 0 \implies \dots \quad (6 \text{ puan} + 8)$$