

26/01/2026

A6 | Να λύσετε τις εξισώσεις

6ε) 88 i) $(\sqrt{3} + \epsilon\phi x)(1 - \epsilon\phi x) = 0$

ii) $(2\sigma\upsilon\nu x + 1)(\epsilon\phi^2 x - 3)\sigma\phi x = 0$

ii) $(2\sigma\upsilon\nu x + 1)(\epsilon\phi^2 x - 3)\sigma\phi x = 0 \xrightarrow[x \neq k\pi, k \in \mathbb{Z}]{x \neq k\pi + \frac{\pi}{2}}$

$\sigma\upsilon\nu x = 0 \Rightarrow x = k\pi + \frac{\pi}{2}$

$\eta\mu x = 0 \Rightarrow x = k\pi$

$(2\sigma\upsilon\nu x + 1)(\epsilon\phi x - \sqrt{3})(\epsilon\phi x + \sqrt{3})\sigma\phi x = 0 \Rightarrow$

$\sigma\upsilon\nu x = -\frac{1}{2} \text{ ή } \epsilon\phi x = \sqrt{3} \text{ ή } \epsilon\phi x = -\sqrt{3} \text{ ή } \sigma\phi x = 0 \Rightarrow$

$\sigma\upsilon\nu x = -\sigma\upsilon\nu \frac{\pi}{3} \text{ ή } \epsilon\phi x = \epsilon\phi \frac{\pi}{3} \text{ ή } \epsilon\phi x = -\epsilon\phi \frac{\pi}{3} \text{ ή } \sigma\phi x = \sigma\phi \frac{\pi}{2} \Rightarrow$

$\sigma\upsilon\nu x = \sigma\upsilon\nu \frac{2\pi}{3} \text{ ή } \epsilon\phi x = \epsilon\phi \frac{\pi}{3} \text{ ή } \epsilon\phi x = \epsilon\phi(-\frac{\pi}{3}) \text{ ή } \sigma\phi x = \sigma\phi \frac{\pi}{2} \Rightarrow$

$x = 2k\pi + \frac{2\pi}{3} \text{ ή } x = 2k\pi - \frac{2\pi}{3} \text{ ή } x = k\pi + \frac{\pi}{3} \text{ ή } x = k\pi - \frac{\pi}{3} \text{ ή } x = k\pi + \frac{\pi}{2} \Rightarrow$

$x = k\pi + \frac{\pi}{3} \text{ ή } x = k\pi - \frac{\pi}{3}, k \in \mathbb{Z}. \quad (\alpha\pi\omicron\rho.)$

A8 | Να λύσετε τις εξισώσεις

6ε) 88 i) $2 \cdot \eta\mu 3x = \sqrt{3}$

ii) $\sigma\upsilon\nu \frac{x}{5} + 1 = 0$

iii) $3\epsilon\phi \frac{2x}{7} - \sqrt{3} = 0$

i) $2\eta\mu 3x = \sqrt{3} \Rightarrow$

$\eta\mu 3x = \frac{\sqrt{3}}{2} \Rightarrow$

$\eta\mu 3x = \eta\mu \frac{\pi}{3} \Rightarrow$

$3x = 2k\pi + \frac{\pi}{3} \text{ ή } 3x = 2k\pi + \frac{2\pi}{3} \Rightarrow$

$x = \frac{2k\pi}{3} + \frac{\pi}{9} \text{ ή } x = \frac{2k\pi}{3} + \frac{2\pi}{9}, k \in \mathbb{Z}.$

ii) $\sigma\upsilon\nu \frac{x}{5} + 1 = 0 \Rightarrow$

$\sigma\upsilon\nu \frac{x}{5} = -1 \Rightarrow$

$\frac{x}{5} = 2k\pi + \pi \Rightarrow$

$x = 10k\pi + 5\pi, k \in \mathbb{Z}.$

$$\text{ii)} \quad 3\epsilon\phi\frac{2x}{7} - \sqrt{3} = 0 \implies$$

$$\begin{aligned} \epsilon\phi\frac{2x}{7} &= \frac{\sqrt{3}}{3} \implies \\ \epsilon\phi\frac{2x}{7} &= \epsilon\phi\frac{\pi}{6} \implies \\ \frac{2x}{7} &= k\pi + \frac{\pi}{6} \implies \\ x &= \frac{7k\pi}{2} + \frac{7\pi}{12}, \quad k \in \mathbb{Z}. \end{aligned}$$

$$\begin{aligned} \neg \text{πειρα } 6\pi\frac{2x}{7} \neq 0 &\implies \\ \frac{2x}{7} &\neq k\pi + \frac{\pi}{2} \implies \\ x &\neq \frac{7k\pi}{2} + \frac{7\pi}{4}, \quad k \in \mathbb{Z} \end{aligned}$$

10. Να λύσετε τις εξισώσεις

i) $2\eta\mu^2\omega + \eta\mu\omega - 1 = 0$

ii) $2\sigma\upsilon\nu^2 x + 3\sigma\upsilon\nu x - 2 = 0$ iii) $3\epsilon\phi^2 t = 3 + 2\sqrt{3}\epsilon\phi t$

$$\text{ii)} \quad 2\sigma\upsilon\nu^2 x + 3\sigma\upsilon\nu x - 2 = 0$$

$$\Delta = 3^2 - 4 \cdot 2 \cdot (-2) = 9 + 16 = 25 > 0$$

$$\sigma\upsilon\nu x = \frac{-3 \pm 5}{4} = \begin{cases} \frac{1}{2} \\ -2 \end{cases}$$

$$\text{Αρα } \sigma\upsilon\nu x = \frac{1}{2} \quad \text{ή} \quad \sigma\upsilon\nu x = -2 \implies$$

(αδύνατον)

$$\eta\mu x, \sigma\upsilon\nu x \in [-1, 1] \quad \forall x \in \mathbb{R}$$

$$\sigma\upsilon\nu x = \sigma\upsilon\nu\frac{\pi}{3} \implies$$

$$x = 2k\pi + \frac{\pi}{3} \quad \text{ή} \quad x = 2k\pi - \frac{\pi}{3}, \quad k \in \mathbb{Z}.$$

$$\text{ii)} \quad 3\epsilon\phi^2 t = 3 + 2\sqrt{3}\epsilon\phi t \xrightarrow{t \neq k\pi + \frac{\pi}{2}, k \in \mathbb{Z}}$$

$$3\epsilon\phi^2 t - 2\sqrt{3}\epsilon\phi t - 3 = 0$$

$$\Delta = (-2\sqrt{3})^2 - 4 \cdot 3 \cdot (-3) = 12 + 36 = 48 > 0$$

$$\epsilon\phi t = \frac{-(-2\sqrt{3}) \pm 4\sqrt{3}}{2 \cdot 3} = \frac{2\sqrt{3} \pm 4\sqrt{3}}{6} = \begin{cases} \sqrt{3} \\ -\frac{\sqrt{3}}{3} \end{cases}$$

$$\text{Αρα } \epsilon\phi x = \sqrt{3} \quad \text{ή} \quad \epsilon\phi x = -\frac{\sqrt{3}}{3} \implies$$

$$\epsilon\phi x = \epsilon\phi\frac{\pi}{3} \quad \text{ή} \quad \epsilon\phi x = \epsilon\phi\left(-\frac{\pi}{6}\right) \implies$$

$$x = k\pi + \frac{\pi}{3} \quad \text{ή} \quad x = k\pi - \frac{\pi}{6}, \quad k \in \mathbb{Z}.$$

11. Να λύσετε τις εξισώσεις

i) $\eta\mu^2 x + 5\sigma\upsilon\nu^2 x = 4$

ii) $\epsilon\phi x \cdot \sigma\phi 2x = 1$

ii) $\epsilon\phi x \cdot \sigma\phi 2x = 1 \xrightarrow{x \neq \frac{k\pi}{2}, k \in \mathbb{Z}}$
 $\sigma\phi 2x = \frac{1}{\epsilon\phi x} \Rightarrow$
 $\sigma\phi 2x = \sigma\phi x \Rightarrow$
 $2x = k\pi + x \Rightarrow$
 $x = k\pi, k \in \mathbb{Z}.$
(σολογ.)

Άρα η εξίσωση είναι δύνατη.

$\sigma\upsilon\nu x \neq 0 \Rightarrow x \neq k\pi + \frac{\pi}{2}$
 $\eta\mu 2x \neq 0 \Rightarrow 2x \neq k\pi \Rightarrow$
 $\Rightarrow x \neq \frac{k\pi}{2}, k \in \mathbb{Z}$