

28/01/2026

Πρόβλημα ζριώνυμο

$$ax^2 + \beta x + \gamma, a \neq 0$$

$$\Delta = \beta^2 - 4\alpha\gamma$$

• $\Delta > 0$

$$x_{1,2} = \frac{-\beta \pm \sqrt{\Delta}}{2a}$$

$a > 0$

x	$-\infty$	x_1	x_2	$+\infty$
$ax^2 + \beta x + \gamma$	+	⊖	⊖	+

$a < 0$

x	$-\infty$	x_1	x_2	$+\infty$
$ax^2 + \beta x + \gamma$	-	⊖	+	-

• $\Delta = 0$

$$x_0 = \frac{-\beta}{2a}$$

$a > 0$

x	$-\infty$	x_0	$+\infty$
$ax^2 + \beta x + \gamma$	+	⊕	+

$a < 0$

x	$-\infty$	x_0	$+\infty$
$ax^2 + \beta x + \gamma$	-	⊕	-

• $\Delta < 0$

$a > 0$

x	$-\infty$	$+\infty$
$ax^2 + \beta x + \gamma$	+	+

$a < 0$

x	$-\infty$	$+\infty$
$ax^2 + \beta x + \gamma$	-	-

A5 | Να λύσετε τις ανισώσεις:

62/113 i) $5x^2 \leq 20x$

ii) $x^2 + 3x \leq 4$.

$$i) 5x^2 \leq 20x \Rightarrow x^2 - 4x \leq 0 \Rightarrow x \in [0, 4]$$

$$\begin{aligned} \bullet x^2 - 4x &= 0 \Rightarrow x(x-4) = 0 \Rightarrow \\ x &= 0 \quad \vee \quad x = 4 \end{aligned}$$

x	$-\infty$	0	4	$+\infty$
$x^2 - 4x$	+	0	0	+

$$ii) x^2 + 3x \leq 4 \Rightarrow x^2 + 3x - 4 \leq 0$$

$$\begin{aligned} \Delta &= 3^2 - 4 \cdot (-4) = 9 + 16 = 25 > 0 \\ x_{1,2} &= \frac{-3 \pm \sqrt{25}}{2 \cdot 1} = \frac{-3 \pm 5}{2} = \begin{cases} 1 \\ -4 \end{cases} \end{aligned}$$

$$\text{Άρα } x \in [-4, 1].$$

x	$-\infty$	-4	1	$+\infty$
$x^2 + 3x - 4$	+	0	0	+

Α 6. Να λύσετε τις ανισώσεις:
6α) 13 i) $x^2 - x - 2 > 0$

$$ii) 2x^2 - 3x - 5 < 0.$$

$$i) x^2 - x - 2 > 0$$

$$\begin{aligned} \Delta &= (-1)^2 - 4 \cdot (-2) = 1 + 8 = 9 > 0 \\ x_{1,2} &= \frac{-(-1) \pm \sqrt{9}}{2 \cdot 1} = \frac{1 \pm 3}{2} = \begin{cases} 2 \\ -1 \end{cases} \end{aligned}$$

$$\text{Άρα } x \in (-\infty, -1) \cup (2, +\infty).$$

x	$-\infty$	-1	2	$+\infty$
$x^2 - x - 2$	+	0	0	+

$$ii) 2x^2 - 3x - 5 < 0$$

$$\begin{aligned} \Delta &= (-3)^2 - 4 \cdot 2 \cdot (-5) = 9 + 40 = 49 > 0 \\ x_{1,2} &= \frac{-(-3) \pm \sqrt{49}}{2 \cdot 2} = \frac{3 \pm 7}{4} = \begin{cases} \frac{5}{2} \\ -1 \end{cases} \end{aligned}$$

$$\text{Άρα } x \in (-1, \frac{5}{2})$$

x	$-\infty$	-1	$\frac{5}{2}$	$+\infty$
$2x^2 - 3x - 5$	+	0	0	+

A7. Να λύσετε τις ανισώσεις:
 6ε).113 i) $x^2 + 4 > 4x$

ii) $x^2 + 9 \leq 6x$.

i) $x^2 + 4 > 4x \Rightarrow$
 $x^2 - 4x + 4 > 0$

$$\Delta = (-4)^2 - 4 \cdot 4 = 16 - 16 = 0$$

$$x_0 = \frac{-(-4)}{2 \cdot 1} = \frac{4}{2} = 2$$

Άρα $x \in (-\infty, 2) \cup (2, +\infty)$

x	$-\infty$	2	$+\infty$
$x^2 - 4x + 4$	+	0	+

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$$x^2 + 4 > 4x \Rightarrow$$

$$x^2 - 4x + 4 > 0 \Rightarrow$$

$$(x-2)^2 > 0 \xRightarrow{(x-2)^2 \geq 0}$$

$$(x-2)^2 \neq 0 \Rightarrow$$

$$x-2 \neq 0 \Rightarrow$$

$$x \neq 2$$

ii) $x^2 + 9 \leq 6x \Rightarrow$
 $x^2 - 6x + 9 \leq 0$

$$\Delta = (-6)^2 - 4 \cdot 9 = 36 - 36 = 0$$

$$x_0 = \frac{-(-6)}{2 \cdot 1} = \frac{6}{2} = 3$$

Άρα $x = 3$

x	$-\infty$	3	$+\infty$
$x^2 - 6x + 9$	+	0	+

A8. Να λύσετε τις ανισώσεις:
 6ε).113 i) $x^2 + 3x + 5 \leq 0$

ii) $2x^2 - 3x + 20 > 0$.

i) $x^2 + 3x + 5 \leq 0$

Αδύνατον

$$\Delta = 3^2 - 4 \cdot 5 = 9 - 20 = -11 < 0$$

x	$-\infty$	$+\infty$
$x^2 + 3x + 5$	+	+

$$ii) \quad 2x^2 - 3x + 20 > 0$$

↑
 Approxim (in $x \in \mathbb{R}$)

$$\Delta = (-3)^2 - 4 \cdot 2 \cdot 20 =$$

$$= 9 - 160 = -151 < 0$$

x	$-\infty$	$+\infty$
$2x^2 - 3x + 20$		
	+	+